CBCS SCHEME

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21MAT21

Second Semester B.E. Degree Examination, June/July 2024
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$. **(06 Marks)**

b. Evaluate $\iint_R xy\,dx\,dy$, where R is bounded by x = 2a, the curve $x^2 = 4ay$. **(07 Marks)**

c. Prove that $\beta(m,n) = \frac{\overline{|m}\,\overline{|n}}{\overline{|m+n}}$ **(07 Marks)**

OR

2 a. Evaluate $\int_{0}^{a}\int_{y}^{a}\frac{x}{x^2+y^2}\,dx\,dy$ by changing the order of integration. **(06 Marks)**

b. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. **(07 Marks)**

c. Prove that $\int_{0}^{\frac{\pi}{2}}\sqrt{\sin\theta}\,d\theta \times \int_{0}^{\frac{\pi}{2}}\frac{1}{\sqrt{\sin\theta}}\,d\theta = \pi$. **(07 Marks)**

Module-2

3 a. Find the directional derivative of $Q = xy^3 + yz^2$ at the point (2, -1, 1) in the direction of $i + 2j + 2k$. **(06 Marks)**

b. Find divF and curlF, where $F = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. Show that $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2+y^2}$ is both solenoidal and irrotational. **(07 Marks)**

OR

4 a. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line joining (0, 0, 0) to (2, 1, 3). **(06 Marks)**

b. Apply Green's theorem to evaluate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region bounded by x = 0, y = 0 and x + y = 1 **(07 Marks)**

c. Using stoke's theorem evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and C is the boundary of the rectangle $x = \pm a$, y = 0, y = b. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by the elimination of arbitrary functions from, $z = f(x+ay) + g(x-ay)$. (06 Marks)

b. Derive the one dimensional heat equation. (07 Marks)

c. Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$. (07 Marks)

OR

6 a. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0, when y is an odd multiple of $\frac{\pi}{2}$. (06 Marks)

b. Form the partial differential equations from $f(x+y+z,\ x^2+y^2+z^2) = 0$. (07 Marks)

c. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$. (07 Marks)

Module-4

7 a. Find a real root of the equation $x\log_{10} x = 1.2$ by regula-falsi method, correct to four decimal places. (06 Marks)

b. Find the cubic polynomial which takes the following values by using Newton's Forward interpolation.

x	0	1	2	3
f(x)	1	2	1	10

and hence evaluate f(4). (07 Marks)

c. Evaluate $\int_0^6 \frac{dx}{1+x^2}$, by using (i) Simpson's rule (ii) Simpson's $\left(\frac{3}{8}\right)^{th}$ rule. (07 Marks)

OR

8 a. Using Newton's-Raphson method find real root of the equation, $3x - \cos x - 1 = 0$ near x = 0.5, correct to 3 decimal places. (06 Marks)

b. Using Newton's divided difference formula for the following data :

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Evaluate f(9). (07 Marks)

c. From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks :	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students :	31	42	51	35	31

(07 Marks)

Module-5

9 a. Using Taylor's series method, solve $\frac{dy}{dx} = 2y + 3e^x$, find y(0.2) with y(0) = 0 upto 4^{th} order derivative with expansion. (06 Marks)

b. Use Runge-Kutta method to find an approximate value of y when x = 0.2 given that $\frac{dy}{dx} = x + y^2$ with y(0) = 1 and taking h = 0.2. (07 Marks)

c. Apply Milne's Predictor-corrector method, find y at x = 0.8 given $\frac{dy}{dx} = x - y^2$ with y(0) = 0, h(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762 and h = 0.2. (07 Marks)

OR

10 a. Using Taylor's series method, find the value of y at x = 0.1 from $\frac{dy}{dx} = x^2y - 1$, y(0) = 1, upto 4th order derivative in the expansion. **(06 Marks)**

b. Apply modified Euler's method, find y(0.2) and h = 0.2 given $\frac{dy}{dx} = x + \sqrt{|y|}$, with y(0) = 1. Carry out two iterations. **(07 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.04 and y(0.3) = 2.09. Find y(0.4) by using Milne's predictor-corrector method. **(07 Marks)**

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